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SR No - 2764/2022

Name of the Course : B. A. (Prog.)
Unique Paper Code : 62357603
Name of the Paper : Numerical Methods
Semester : VI
Duration : 3 Hours
Maximum Marks : 75 Marks
Duration: 3 hours

(Write your roll no. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any two parts from each question.
Use of Non – Programmable Scientific Calculator is allowed.

Q-1. (a) Find the root of the equation $f(x) = \cos x - xe^x$ lying between (0,1) by the Bisection method. [6]

(b) Obtain the rate of Convergence of Regula - Falsi method. [6]

(c) Suppose 1.414 is used as an approximation to $\sqrt{2}$, then find the absolute and relative error. [6]

(d) Find a real root of the equation given by $x^2 - 7x + 1 = 0$ by Newton – Raphson Method. [6]

Q-2. (a) Solve the given equation $f(x) = x^3 + x^2 + x + 6$ by Secant method. [6.5]

(b) Perform two iterations of Newton's method to solve the non – linear system of equation with initial approximation (1,1) :-

$$f(x,y) = x^2 + y^2 - 4 = 0,$$
$$g(x,y) = x^2 + y^2 - 16 = 0$$ [6.5]

(c) Round off the number 784320 to four significant digits and compute Ea (Absolute error), Er (Relative error) and Ep (Percentage error) for this number. [6.5]

(d) Find the real root of the equation $f(x) = x^3 - 9x + 2 = 0$ by Regula – Falsi method in the interval 2 and 3. [6.5]

Q-3. (a) Starting with initial vector $(x, y, z) = (0, 0, 0)$, perform three iterations of Gauss-Seidel method to solve the following system of equation:

$$2x - y = 7$$
$$-x + 2y - z = 1$$
$$-y + 2z = 1.$$ [6]

(b) Construct the interpolating polynomial using the Gregory-Newton backward difference interpolation for the given data set: [6]

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.4	1.5	1.7	2.0	2.2

Hence estimate the value of $f(x)$ at $x = 0.35$.

(c) Using Gauss-Jordan method, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$ [6]

(d) Consider the following table: [6]

x	1	2	4	5
$f(x)$	5.3	2.0	3.1	1.0

Use dividend difference to calculate the interpolating polynomial of degree 3 and give an estimate for $f(1.5)$.

Q-4. (a) Find the Lagrange interpolation polynomial for the data set: $(0, 1), (1, 3)$ and $(3, 5.5)$. Also estimate the value at $x = 1.5$. [6.5]

(b) Using Gaussian Elimination method, solve the following system of equations:

$$4x - 3y + z = -8$$
$$-2x + y - 3z = -4$$
$$x - y + 2z = 3.$$

[6.5]

(c) For the following system of equations:

$$4x + y + z = 2$$
$$x + 5y + 2z = -6$$
$$x + 2y + 3z = -4.$$

Use Gauss-Jacobi iteration method by performing three iterations. Take the initial approximation as $(x, y, z) = (1,1,1)$. [6.5]

(d) Prove the following relation:

$$\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}.$$

[6.5]

Also construct the forward difference table at the following data points:

x	0	1	2	3	4
$f(x)$	1	7	23	55	109

Q-5. (a) The velocities of a car running on a straight road at intervals of 2 minutes are given below:

Time(min)	0	2	4	6	8	10	12
Velocity(km/hr)	0	22	30	27	18	7	0

Using Simpson $\frac{1}{3}$rd rule find the distance covered by the car. [6]

(b) Given that $\frac{dy}{dx} - \sqrt{xy} = 2$ with $y(1) = 1$, find $y(2)$ by Euler's method (Take $h = 0.5$). [6]

(c) Using Midpoint method obtain the numerical solution $y(0.2)$ of the initial value problem $y' = (x - y)^2; y(0) = 0.5$ correct to 4 decimal places, with $h = 0.1$. [6]

(d) Calculate the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 10$; given the following table:- [6]

x	3	5	11	27
f	-13	23	89	170

Q-6. (a) Evaluate $\int_0^1 \frac{1+x}{1+x^3} dx$ using Trapezoidal rule by dividing interval into five equal parts. [6.5]

(b) Given that the differential equation $\frac{dy}{dx} = x^2 + y, y(0) = 1$. Find the value of $y(0.02)$, using Euler's modified method. [6.5]

(c) Find $f'(x_2)$ using central difference formula and Richardson extrapolation with $h = 2$, $(x_0, y_0) = (1,2)$, $(x_1, y_1) = (2,4)$, $(x_2, y_2) = (3,8)$, $(x_3, y_3) = (4,16)$, $(x_4, y_4) = (5,32)$. [6.5]

(d) Find the value of the integral $I = \int_0^4 \frac{dx}{1+x^2}$ using the Simpson 3/8th rule with $h = 1$. [6.5]

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[This question paper contains 4 printed pages.]

Your Roll No. 2022

Sr. No. of Question Paper : 2765 **A**

Unique Paper Code : 62357604

Name of the Paper : Differential Equations

Name of the Course : **B.A. (Prog.) Mathematics : DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1.

a. Solve : 6

$$(8ydx + 8xdy) + x^2y^3(4ydx + 5xdy) = 0.$$

b. Solve : 6

$$(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0.$$

c. Solve : 6

$$p^2 + 2p\,y\,cot(x) - y^2 = 0.$$

d. Show that $1, e^x, e^{2x}$ are the linearly independent solutions of 6

$$y''' - 3y'' + 2y' = 0.$$

What is the general solution? Find the solution y(x) with the property $y(0) = 0$, $y'(0) = 2$, $y''(0) = 3$.

2.

a. Solve : 6.5

$$y' + y\cos(x) = y^2 \sin(2x).$$

b. Solve : 6.5

$$y = p^3 + p^2 x.$$

c. Convert into Clairaut's form and hence solve :

$$y = 2xp + xp^2.$$

d. Show that e^{-2x}, xe^{-2x} are the linearly independent solutions of 6.5

$$y'' + 4y' + 4y = 0.$$

What is the general solution? Find the solution y(x) with the property $y(0) = 0$, $y'(0) = 2$.

3.

a. Solve: 6

$$\left(\frac{d^2y}{dx^2}\right) + 2\left(\frac{dy}{dx}\right) + 10y + 37sin3x = 0.$$

b. Solve: 6

$$(x^2D^2 + 7xD + 13)y = \log x.$$

c. Apply the method of variation of parameter to solve: 6

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}.$$

d. Find the solution of : 6

$$\left(\frac{d^2y}{dx^2}\right) + 4y = 8\,cos2x$$

given that $y = 0$ and $\frac{dy}{dx} = 0$, when $x = 0$.

4.

a. Solve the following system of equations: 6.5

$$\frac{dx}{dt} - y = t \text{ and } \frac{dy}{dt} + x = 1.$$

b. Solve : 6.5

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}.$$

c. Solve : 6.5

$$(ydx + xdy)(a - z) + xy\,dz = 0.$$

d. Solve : 6.5

$$\frac{dx}{-xy^2} = \frac{dy}{y^3} = \frac{dz}{axz}.$$

5.

a. Eliminate the arbitrary function f from the equation: 6

$$x + y + z = f(x^2 + y^2 + z^2)$$

to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6

$$x^2p + y^2q = (x + y)z.$$

c. Find the integral surface of the linear partial differential equation 6

$$(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$$

which contains the straight line $y = 0$ and $x = 1$.

d. Find the complete integral of the partial differential equation: 6

$$q = (z + px)^2.$$

6.

a. (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic: 2.5

$$x^2(y - 1)r - x(y^2 - 1)s + y(y - 1)t + xyp - q = 0$$

where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

(ii) Eliminate the arbitrary constants a and b from the equation 4

$$ax^2 + by^2 + z^2 = 1$$

to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6.5

$$y^2(x - y)p + x^2(y - x)q = z(x^2 + y^2).$$

c. Show that the following systems of partial differential equations are compatible and hence solve them 6.5

$$p = y\left(1 + \frac{1}{x}\right) + \cos y, \quad q = x + \log x - x \sin y.$$

d. Find the complete integral of the partial differential equation: 6.5

$$z = p^2 - q^2 .$$

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[This question paper contains 3 printed pages.]

Your Roll No. 2022

Sr. No. of Question Paper : 2780 **A**

Unique Paper Code : 62357604

Name of the Paper : Differential Equations

Name of the Course : **B.A. (Prog.) Mathematics : DSE**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1.

a. Solve : 6

$$(1 + yx)xdy + (1 - yx)ydx = 0$$

b. Solve : 6

$$(y^3 - x^2y)dx + \left(3xy^2 - \frac{x^3}{3}\right)dy = 0$$

c. Solve : 6

$$x + yp^2 - p(1 + xy) = 0$$

d. Show that e^{-2x}, e^{-3x} are the linearly independent solutions of 6

$$y'' + 5y' + 6y = 0.$$

What is the general solution? Find the solution y(x) with the property $y(0) = 0$, $y'(0) = 1$.

P.T.O.

2.

a. Solve :

$$x^3y' - x^2y = -y^4\sin(x).$$ 6.5

b. Solve : 6.5

$$4y = x^2 + p^2.$$

c. Solve : 6.5

$$xy(y - px) = (x + py)$$

d. Show that e^{-x}, xe^{-x}, e^{2x} are the linearly independent solutions of 6.5

$$y''' - 3y' - 2y = 0.$$

What is the general solution? Find the solution y(x) with the property $y(0) = 0$, $y'(0) = 2, y''(0) = 3$.

3.

a. Solve : 6

$$\left(\frac{d^3y}{dx^3}\right) - 3\left(\frac{d^2y}{dx^2}\right) - 6\left(\frac{dy}{dx}\right) = x^2 + 1.$$

b. Solve : 6

$$(x^2D^2 - 3xD + 5)y = x^2 \sin\log x.$$

c. Apply the method of variation of parameter to solve : 6

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\,e^x\log x, \quad x > 0.$$

d. Solve : 6

$$(D^2 + a^2)y = \cos ax.$$

4.

a. Solve the following system of equations : 6.5

$$\frac{dx}{dt} + 2y + x = e^t \text{ and } \frac{dy}{dt} + 2x + y = 3e^t.$$

b. Solve : 6.5

$$\frac{dx}{x^2+2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}.$$

c. Solve : 6.5

$$xz^3dx - zdy + 2y\,dz = 0.$$

d. Solve : 6.5

$$\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}.$$

5.

a. Eliminate the arbitrary function f from the equation: 6

$$z = e^{ax+by} f(ax - by)$$

to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6

$$(x^2 + 2y^2)p - xyq = xz.$$

c. Find the integral surface of the linear partial differential equation 6

$$(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$$

Through the curve $xz=a^3$ and $y=0$.

d. Find the complete integral of the partial differential equation: 6

$$z=px+qy+p^2+q^2.$$

6.

a. (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic: 2.5

$$(x-y)(xr-xs-ys+yt)=(x+y)(p-q),$$

where $r=\frac{\partial^2 z}{\partial x^2}$, $s=\frac{\partial^2 z}{\partial x\,\partial y}$, $t=\frac{\partial^2 z}{\partial y^2}$, $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$.

(ii) Eliminate the arbitrary constants a and b from the equation : 4

$$2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$

to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6.5

$$px(z-2y^2)=(z-qy)(z-y^2-2x^3).$$

c. Show that the following systems of partial differential equations are compatible and hence solve them 6.5

$$p=x^4-2xy^2+y^4, \qquad q=4xy^3-2x^2y-\sin y.$$

d. Find the complete integral of the partial differential equation: 6.5

$$zpq=p^2(xq+p^2)+q^2(yp+q^2).$$